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Nonequilibrium thermodynamic study of magnetization dynamics in the presence of spin-transfer torque

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The dynamics of magnetization in the presence of spin-transfer torque was studied. We derived the equation for the motion of magnetization in the presence of a spin current by using the local equilibrium assumption in nonequilibrium thermodynamics. We show that, in the resultant equation, the ratio of the Gilbert damping constant, α, and the coefficient, β, of the current-induced torque called nonadiabatic torque, depends on the relaxation time of the fluctuating field τ_c. The equality $\alpha=\beta$ holds when τ_c is very short compared to the time scale of magnetization dynamics. We apply our theory to current-induced magnetization reversal in magnetic multilayers and show that the switching time is a decreasing function of τ_c.

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Spin-transfer torque-induced magnetization dynamics such as current-induced magnetization reversal,[1–3] domain-wall motion,[4] and microwave generation[5] have attracted a great deal of attention because of their potential applications to future nanospinelectronic devices. In the absence of spin-transfer torque, magnetization dynamics is described by either the Landau-Lifshitz (LL) equation[6] or the Landau-Lifshitz-Gilbert (LLG) equation.[7] It is known that the LL and LLG equations become equivalent through rescaling of the gyromagnetic ratio. However, this is not the case in the presence of spin-transfer torque. For domain-wall dynamics, the following LLG-type equation has been studied by several groups:[8–12]

$$\partial_t\langle \boldsymbol{M}\rangle + \boldsymbol{v}\cdot\boldsymbol{\nabla}\langle \boldsymbol{M}\rangle = \gamma \boldsymbol{H}\times\langle \boldsymbol{M}\rangle + \frac{\alpha}{M}\langle \boldsymbol{M}\rangle\times\partial_t\langle \boldsymbol{M}\rangle + \frac{\beta}{M}\langle \boldsymbol{M}\rangle \times[(\boldsymbol{v}\cdot\boldsymbol{\nabla})\langle \boldsymbol{M}\rangle], \qquad (1)$$

where $\boldsymbol{M}$ represents the magnetization, $\boldsymbol{v}$ is the velocity, γ is the gyromagnetic ratio, and α is the Gilbert damping constant. The second term on the left-hand side represents the adiabatic contribution of spin-transfer torque. The first and the second terms on the right-hand side are the torque due to the effective magnetic field $\boldsymbol{H}$ and the Gilbert damping. The last term on the right-hand side of Eq. (1) represents the current-induced torque called "nonadiabatic torque" or simply the β term. The directions of the adiabatic contribution of spin-transfer torque and nonadiabatic torque are shown in Fig. 1(a).

As shown by Thiaville *et al.*, the value of the coefficient β strongly influences the motion of the domain wall.[8] However, the value of the coefficient β is still controversial, and different conclusions have been drawn from different approaches.[11–18] For example, Barnes and Maekawa showed that the value of β should be equal to that of the Gilbert damping constant α to satisfy the requirement that the relaxation should cease at the minimum of electrostatic energy, even under particle flow. Kohno *et al.*[12] performed microscopic calculations of spin torques in disordered ferromagnets and showed that the α and β terms arise from the spin-relaxation processes and that $\alpha\neq\beta$ in general. Tserkovnyak *et al.*[13] derived the β term using a quasiparticle approximation and showed that $\alpha=\beta$ within a self-consistent picture based on the local-density approximation.

In the current-induced magnetization dynamics in the magnetic multilayers shown in Fig. 1(b),[19–21] the nonadiabatic torque exerts a strong effect and therefore affects the direct current voltage of the spin torque diode, as shown in Refs. 20 and 21. The magnetization dynamics of the free layer, $\boldsymbol{S}_2$, has been studied by using the following LLG-type equation:

$$\partial_t\boldsymbol{S}_2 - \frac{I}{e}g\hbar(\boldsymbol{S}_2\times\boldsymbol{S}_1)\times\boldsymbol{S}_2 = \gamma\boldsymbol{H}\times\boldsymbol{S}_2 + \frac{\alpha}{S_2}\boldsymbol{S}_2 \times\partial_t\boldsymbol{S}_2 + \eta I\boldsymbol{S}_2\times\boldsymbol{S}_1, \qquad (2)$$

where I is the charge current density, g is the amplitude of the spin torque introduced by Slonczewski,[1] $\hbar$ is the Dirac constant, and η is the magnitude of the nonadiabatic torque, which is sometimes called the fieldlike torque.[20,21]

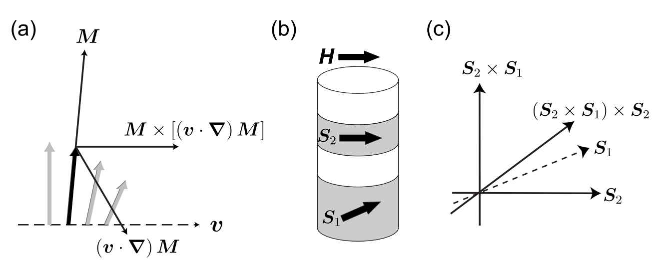

FIG. 1. (a) The direction of the magnetization $\boldsymbol{M}$, the adiabatic contribution of spin-transfer torque, $(\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{M}$, and the β term, $\boldsymbol{M}\times[(\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{M}]$, are shown. The direction of the velocity $\boldsymbol{v}$ is indicated by the dotted arrow. (b) The magnetic multilayers, in which the pinned and the free layers are separated by a nonmagnetic spacer layer are schematically shown. The magnetization vectors of the pinned and free layers are represented by $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$, respectively. The effective magnetic field, to which $\boldsymbol{S}_2$ is subject, is represented by $\boldsymbol{H}$. (c) The direction of the magnetization of the free layer, $\boldsymbol{S}_2$, the spin-transfer torque $(\boldsymbol{S}_2\times\boldsymbol{S}_1)\times\boldsymbol{S}_2$, and the nonadiabatic torque, $\boldsymbol{S}_2\times\boldsymbol{S}_1$, are shown. The direction of $\boldsymbol{S}_1$ is indicated by the dotted arrow.

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In this Rapid Communication, we study the magnetization dynamics induced by spin-transfer torque in the framework of nonequilibrium thermodynamics. We derive the equation of motion of the magnetization in the presence of a spin current by using the local equilibrium assumption. In the resultant equation, the Gilbert damping term and the β term are expressed as memory terms with the relaxation time of the fluctuating field τ_c. We show that the value of the coefficient β is not equal to that of the Gilbert damping constant α in general. However, we also show that the equality $\alpha=\beta$ holds if $\tau_c \ll 1/(\gamma H)$. We apply our theory to the current-induced magnetization reversal in magnetic multilayers and show that the switching time is a decreasing function of τ_c.

Let us first briefly introduce the nonequilibrium statistical theory of magnetization dynamics in the absence of spin current.[22] The LLG equation describing the motion of magnetization $\boldsymbol{M}$ under an effective magnetic field $\boldsymbol{H}$ is given by

$$\partial_t \boldsymbol{M} = \gamma \boldsymbol{H} \times \boldsymbol{M} + \frac{\alpha}{M} \boldsymbol{M} \times \partial_t \boldsymbol{M}. \tag{3}$$

The equivalent LL equation is expressed as

$$\partial_t \boldsymbol{M} = \frac{\gamma}{1+\alpha^2} \boldsymbol{H} \times \boldsymbol{M} - \frac{\alpha\gamma}{M(1+\alpha^2)} \boldsymbol{M} \times (\boldsymbol{M} \times \boldsymbol{H}). \tag{4}$$

The Langevin equations leading to Eqs. (3) and (4) by taking the ensemble average of magnetization $\boldsymbol{m}$ are

$$\partial_t \boldsymbol{m} = \gamma \boldsymbol{H}_{\rm tot} \times \boldsymbol{m}, \tag{5}$$

$$\partial_t \delta \boldsymbol{H} = -\frac{1}{\tau_c}(\delta \boldsymbol{H} - \chi_s \boldsymbol{m}) + \boldsymbol{R}(t), \tag{6}$$

where the total magnetic field $\boldsymbol{H}_{\rm tot}$ is the sum of the effective magnetic field $\boldsymbol{H}$ and the fluctuating magnetic field $\delta\boldsymbol{H}$, and χ_s is the susceptibility of the local magnetic field induced at the position of the spin. The Fokker-Planck equation corresponding to the Langevin equations guarantees approach to thermal equilibrium.[22] According to Eq. (6) the fluctuating magnetic field $\delta\boldsymbol{H}$ relaxes toward the reaction field $\chi_s \boldsymbol{m}$ with the relaxation time τ_c. The random field $\boldsymbol{R}(t)$ satisfies $\langle \boldsymbol{R}(t)\rangle = \boldsymbol{0}$ and the fluctuation-dissipation relation, $\langle R_i(t) R_j(t')\rangle = \frac{2}{\tau_c}\chi_s k_{\rm B} T \delta_{i,j}\delta(t-t')$, where $k_{\rm B}$ is the Boltzmann constant, T is the temperature, $\langle \cdots \rangle$ is the ensemble average, and $i,j=1,2,3$ are the Cartesian components. It was shown that Eqs. (5) and (6) lead to Kawabata's extended Landau-Lifshitz equation[23] derived by the projection operator method.[22] In the Markovian limit, i.e., $\tau_c \ll 1/(\gamma H)$, we can obtain the LLG Eq. (3) and the corresponding LL Eq. (4) with $\alpha = \gamma \tau_c \chi_s M$.[22] The spin relaxation vanishes in the limit of $\tau_c \to 0$ since it is induced by the transition of spin states during the time τ_c.

In order to consider the flow of spins, i.e., spin current, we introduce the positional dependence. Since we are interested in the average motion, it is convenient to introduce the mean velocity of the carrier, $\boldsymbol{v}$. The average magnetization, $\langle \boldsymbol{m}(\boldsymbol{x},t)\rangle$, is obtained by introducing the positional dependence and taking the ensemble average of Eq. (5). In terms of the mean velocity, the ensemble average of the left-hand side of Eq. (5) leads to $\partial_t \langle \boldsymbol{m}\rangle + (\boldsymbol{v}\cdot\nabla)\langle \boldsymbol{m}\rangle$. Assuming $\langle \delta\boldsymbol{H} \times \boldsymbol{m}\rangle \approx \langle \delta \boldsymbol{H}\rangle \times \langle \boldsymbol{m}\rangle$, which is applicable when the thermal fluctuation is small compared to the mean value, we obtain,

$$\partial_t \langle \boldsymbol{m}\rangle + (\boldsymbol{v}\cdot\nabla)\langle \boldsymbol{m}\rangle = \gamma \langle \boldsymbol{H}_{\rm tot}(\boldsymbol{x},t)\rangle \times \langle \boldsymbol{m}(\boldsymbol{x},t)\rangle. \tag{7}$$

The mean magnetization density is expressed as $\langle \boldsymbol{M}(\boldsymbol{x},t)\rangle = \rho(\boldsymbol{x},t)\langle \boldsymbol{m}(\boldsymbol{x},t)\rangle$, i.e., by the product of the scalar and vectorial components both of which depend on the position of the spin carrier at time t. The spin carrier density, which is the scalar component of spin density, satisfies the continuity equation,

$$\partial_t \rho(\boldsymbol{x},t) + \nabla \cdot [\boldsymbol{v}\rho(\boldsymbol{x},t)] = 0. \tag{8}$$

By multiplying the left-hand side of Eq. (7) by $\rho(\boldsymbol{x},t)$ and by using the continuity Eq. (8), the closed expression for the mean magnetization is obtained as[24]

$$\begin{aligned}\rho(\partial_t \langle \boldsymbol{m}\rangle + \boldsymbol{v}\cdot\nabla\langle \boldsymbol{m}\rangle) &= \partial_t \rho \langle \boldsymbol{m}\rangle + \langle \boldsymbol{m}\rangle \nabla \cdot \boldsymbol{v}\rho + \rho \boldsymbol{v}\cdot\nabla\langle \boldsymbol{m}\rangle \\ &= \partial_t \langle \boldsymbol{M}\rangle + \mathrm{Div}\boldsymbol{v}\langle \boldsymbol{M}\rangle,\end{aligned} \tag{9}$$

where $\mathrm{Div}\boldsymbol{v}\langle \boldsymbol{M}\rangle$ is defined by

$$\mathrm{Div}\boldsymbol{v}\langle \boldsymbol{M}\rangle = \sum_{i=1}^{3} \frac{\partial v_i \langle \boldsymbol{M}\rangle}{\partial x_i} = \langle \boldsymbol{M}\rangle(\nabla\cdot\boldsymbol{v}) + (\boldsymbol{v}\cdot\nabla)\langle \boldsymbol{M}\rangle. \tag{10}$$

By multiplying the right-hand side of Eq. (7) by $\rho(\boldsymbol{x},t)$ and by using Eq. (9), we obtain,

$$\partial_t \langle \boldsymbol{M}\rangle + \mathrm{Div}\boldsymbol{v}\langle \boldsymbol{M}\rangle = \gamma(\boldsymbol{H} + \langle \delta\boldsymbol{H}\rangle) \times \langle \boldsymbol{M}\rangle. \tag{11}$$

Equation (11) takes the standard form of a time evolution equation for extensive thermodynamical variables under flow.[24] The average of Eq. (6) with the positional dependence is given by

$$\partial_t \langle \delta\boldsymbol{H}(\boldsymbol{x},t)\rangle = -\frac{1}{\tau_c}[\langle \delta\boldsymbol{H}(\boldsymbol{x},t)\rangle - \chi\langle \boldsymbol{M}(\boldsymbol{x}(t),t)\rangle], \tag{12}$$

where $\boldsymbol{x}(t)$ is the mean position at time t of the spin carrier, which flows with velocity $\boldsymbol{v} = \partial_t \boldsymbol{x}(t)$. For simplicity, $\chi = \chi_s/\rho$ is assumed to be a constant independent of the position. Equations (11) and (12) constitute the basis for the subsequent study of magnetization dynamics in the presence of spin-transfer torque.

The formal solution of Eq. (12) is expressed as

$$\langle \delta\boldsymbol{H}(\boldsymbol{x},t)\rangle = \frac{\chi}{\tau_c}\int_{-\infty}^{t} \psi(t-t')\langle \boldsymbol{M}[\boldsymbol{x}(t'),t']\rangle dt', \tag{13}$$

where the memory kernel is given by $\psi(t) = \exp[-t/\tau_c]$. Using partial integration, we obtain,

$$\langle \delta\boldsymbol{H}(\boldsymbol{x},t)\rangle = \chi\langle \boldsymbol{M}\rangle - \int_{-\infty}^{t} \psi(t-t')\chi\langle \dot{\boldsymbol{M}}(t')\rangle dt', \tag{14}$$

where the explicit expression for $\dot{\boldsymbol{M}}(t) = \dot{\boldsymbol{M}}[\boldsymbol{x}(t),t]$ is given by the convective derivative,

$$\dot{\boldsymbol{M}}(t) = \partial_t \boldsymbol{M}[\boldsymbol{x}(t),t] + (\boldsymbol{v}\cdot\nabla)\boldsymbol{M}[\boldsymbol{x}(t),t]. \tag{15}$$

Substituting Eq. (14) into Eq. (11), we obtain the equation of motion for the mean magnetization density,

$$\partial_t\langle \boldsymbol{M}\rangle + \mathrm{Div}\boldsymbol{v}\langle \boldsymbol{M}\rangle = \gamma \boldsymbol{H}\times\langle \boldsymbol{M}\rangle + \gamma\int_{-\infty}^{t} dt'\,\psi(t-t')\chi\langle \boldsymbol{M}(t)\rangle \times \langle \dot{\boldsymbol{M}}(t')\rangle. \tag{16}$$

Equation (16) supplemented by Eq. (15) is the *principal result* of this Rapid Communication.

When the relaxation time of the fluctuating field, τ_c, is very short compared to the time scale of the magnetization dynamics, the memory kernel is decoupled and Eq. (16) can be written as

$$\partial_t\langle \boldsymbol{M}\rangle + \mathrm{Div}\boldsymbol{v}\langle \boldsymbol{M}\rangle = \gamma \boldsymbol{H}\times\langle \boldsymbol{M}\rangle + \frac{\alpha}{M}\langle \boldsymbol{M}\rangle \times \langle \dot{\boldsymbol{M}}\rangle, \tag{17}$$

where $\alpha=\gamma\tau_c\chi M$ is the Gilbert damping constant. We substitute the explicit form of the convective derivative, Eq. (15), into Eq. (17). Moreover by using Eq. (10), we obtain the following LLG-type equation:

$$\partial_t\langle \boldsymbol{M}\rangle + \langle \boldsymbol{M}\rangle(\boldsymbol{\nabla}\cdot\boldsymbol{v}) + (\boldsymbol{v}\cdot\boldsymbol{\nabla})\langle \boldsymbol{M}\rangle = \gamma \boldsymbol{H}\times\langle \boldsymbol{M}\rangle + \frac{\alpha}{M}\langle \boldsymbol{M}\rangle \times \partial_t\langle \boldsymbol{M}\rangle + \frac{\alpha}{M}\langle \boldsymbol{M}\rangle \times [(\boldsymbol{v}\cdot\boldsymbol{\nabla})\langle \boldsymbol{M}\rangle]. \tag{18}$$

If $\boldsymbol{\nabla}\cdot\boldsymbol{v}=0$, Eq. (18) reduces to Eq. (14) of Ref. 11, which is derived by replacing the time derivative of magnetization $\partial_t\boldsymbol{M}$ on both sides of the LLG Eq. (3) by the convective derivative $\partial_t\boldsymbol{M}+(\boldsymbol{v}\cdot\boldsymbol{\nabla})\cdot\boldsymbol{M}$. The term $\langle \boldsymbol{M}\rangle(\boldsymbol{\nabla}\cdot\boldsymbol{v})$ appears not on the right-hand side of Eq. (18) but on the left-hand side, which means we cannot obtain Eq. (18) using the same procedure used in Ref. 11. As shown in Refs. 11 and 14, Eq. (18) with $\langle \boldsymbol{M}\rangle(\boldsymbol{\nabla}\cdot\boldsymbol{v})=0$ leads to a steady-state solution in the comoving frame, $\langle \boldsymbol{M}(t)\rangle=\langle \boldsymbol{M}_0(\boldsymbol{x}-\boldsymbol{v}t)\rangle$, where $\langle \boldsymbol{M}_0(\boldsymbol{x})\rangle$ denotes the stationary solution in the absence of domain-wall motion. However, if $\langle \boldsymbol{M}\rangle(\boldsymbol{\nabla}\cdot\boldsymbol{v})\neq 0$, the steady-state solution may break the Galilean invariance. The situation $\langle \boldsymbol{M}\rangle(\boldsymbol{\nabla}\cdot\boldsymbol{v})\neq 0$ can be realized, for example, in magnetic semiconductors,[25,26] where the spin carrier density is spatially inhomogeneous, i.e., $\boldsymbol{\nabla}\rho\neq 0$.

The last term of Eq. (18) represents the nonadiabatic component of the current-induced torque, which is also known as the "β term." By comparing Eq. (18) with Eq. (1), one can see that the coefficient of the last term is equal to the Gilbert damping constant α. However, Eq. (18) is valid when the relaxation time of the fluctuating field, τ_c, is very short compared to the time scale of the magnetization dynamics. It should be noted that the general form of the equation describing the magnetization dynamics is given by Eq. (16) where the last term on the right-hand side is the origin of the α and β terms. It is possible to project the torque represented by the memory function onto the direction of the α and β terms. This projection leads to $\alpha\neq\beta$ in general.

In order to observe the effect of τ_c on the magnetization dynamics we applied our theory to the current-induced magnetization switching in the magnetic multilayer shown in Fig. 1(b). We assumed that the fixed and free layers are single domain magnetic layers acting as a large spin characterized by the total magnetization vector defined as $\boldsymbol{S}_i=\int dV\langle \boldsymbol{M}_i\rangle$, where $i=1(2)$ for the fixed (free) layer and $\int dV$ denotes the volume integration over the fixed (free) layer. Both the magnetization vector of the fixed layer $\boldsymbol{S}_1$ and the effective magnetic field, $\boldsymbol{H}$, acting on the free layer lie in the plane.

Integrating Eqs. (11) and (12) over the volume of the free layer, we obtain the equations:

$$\partial_t\boldsymbol{S}_2 + \int dS\hat{\boldsymbol{n}}\cdot\boldsymbol{J} = \gamma(\boldsymbol{H}+\langle\delta\boldsymbol{H}\rangle)\times\boldsymbol{S}_2, \tag{19}$$

$$\partial_t\langle\delta\boldsymbol{H}\rangle = -\frac{1}{\tau_c}(\langle\delta\boldsymbol{H}\rangle - \chi_V\boldsymbol{S}_2), \tag{20}$$

where $\boldsymbol{J}=\boldsymbol{v}\otimes\langle \boldsymbol{M}\rangle$ is the spin current tensor $\int dS$ represents the surface integration over the free layer, $\hat{\boldsymbol{n}}$ is the unit normal vector of the surface, and $\chi_V=\chi/V$ is defined by the volume of the free layer V.

The same procedure used to derive Eq. (16) yields

$$\partial_t\boldsymbol{S}_2 + \int dS\hat{\boldsymbol{n}}\cdot\boldsymbol{J} = \gamma\boldsymbol{H}\times\boldsymbol{S}_2 + \gamma\int_{-\infty}^{t} dt'\,\psi(t-t')\chi_V\boldsymbol{S}_2(t)\times\partial_{t'}\boldsymbol{S}_2(t'), \tag{21}$$

where $\psi(t)=\exp[-t/\tau_c]$.

When the relaxation time of the fluctuating field is short compared to the time scale of magnetization dynamics, the LLG-type equation in the presence of the spin-transfer torque is obtained as

$$\partial_t\boldsymbol{S}_2 + \int dS\hat{\boldsymbol{n}}\cdot\boldsymbol{J} = \gamma\boldsymbol{H}\times\boldsymbol{S}_2 + \frac{\alpha}{S_2}\boldsymbol{S}_2\times\partial_t\boldsymbol{S}_2, \tag{22}$$

where $\alpha=\gamma\tau_c\chi_V S_2$. By introducing the conventional form of the spin-transfer torque,[1] we obtain the following LLG-type equation:

$$\partial_t\boldsymbol{S}_2 - \frac{I}{e}g\hbar(\boldsymbol{S}_2\times\boldsymbol{S}_1)\times\boldsymbol{S}_2 = \gamma\boldsymbol{H}\times\boldsymbol{S}_2 + \frac{\alpha}{S_2}\boldsymbol{S}_2\times\partial_t\boldsymbol{S}_2. \tag{23}$$

However, Eq. (23) is valid only when $\tau_c < 1/(\gamma H)$. As mentioned before, the torque represented by using the memory function generally has a component parallel to the nonadiabatic torque. In order to observe the effect of the nonadiabatic torque induced by the memory function on the magnetization dynamics, we performed numerical simulation using Eqs. (19) and (20).

For the simulation, we used the following conditions: At the initial time of $t=0$, we assumed that the magnetization of the free layer is aligned parallel to the effective magnetic field $\boldsymbol{H}$ and the angle between the magnetizations of the fixed and the free layers is 45°. This arrangement corresponds to the recent experiment on a magnetic tunnel junction system.[21] We also assumed that the fluctuation field has zero mean value at $t=0$, i.e., $\langle\delta\boldsymbol{H}(0)\rangle=\boldsymbol{0}$.

In Fig. 2(a), we plot the time dependence of the z component of the magnetization of the free layer under the large enough spin current to flip the magnetization of the free layer, $Ig\hbar S_2^2 S_1/(e\alpha\gamma H)=-10$. The value of τ_c is varied and the solid, dotted, and dot-dashed lines correspond to $\gamma H\tau_c=0.1$, 1.0, and 10.0, respectively. As shown in Fig. 2(a),

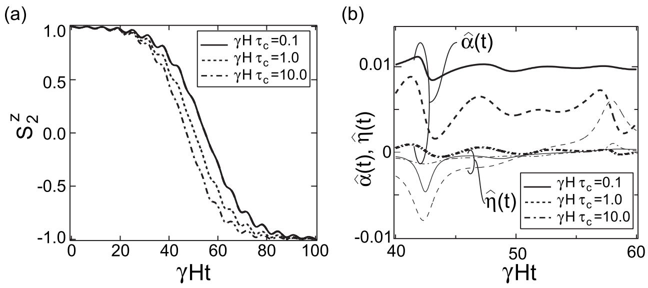

FIG. 2. (a) The z component of the magnetization $\boldsymbol{S}_2$ is plotted against time for various values of τ_c. The initial direction of $\boldsymbol{S}_2$ lies in the direction of the effective magnetic field, which is aligned to the z axis. The initial angle between $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ is taken to be 45°. The value of $\alpha=\gamma\tau_c\chi_s M$ is kept at 0.01. (b) The projections of $\gamma\langle\delta\boldsymbol{H}\rangle\times\boldsymbol{S}_2$ in the direction of $\boldsymbol{S}_2\times\partial_t\boldsymbol{S}_2$ [$\hat{\alpha}(t)$; thick lines] and $\boldsymbol{S}_2\times\boldsymbol{S}_1$ [$\hat{\eta}(t)$; thin lines] are plotted against time during the period of magnetization reversal.

the time required for the magnetization of the free layer to flip decreases with increasing τ_c, which can be understood by projecting the torque given by the last term of Eq. (21) expressed by a memory function. As shown in Fig. 2(b), the Gilbert damping component, $\hat{\alpha}(t)$, decreases from the value 0.01 in the limit of $\tau_c<1/(\gamma H)$ to zero as τ_c increases. The Gilbert damping component delays spin-flip motion. By increasing τ_c, $\hat{\alpha}(t)$ decreases and the time required for $\boldsymbol{S}_2$ to flip decreases. The nonadiabatic component, which is divided by $\gamma H/(IS_1)$ to make it nondimensional is also shown in Fig. 2(b). $\hat{\eta}(t)$ is zero in the limit of $\tau_c<1/(\gamma H)$ and first increases as τ_c increases. The largest nonadiabatic component is found when $\tau_c\approx 1/(\gamma H)$. By further increasing τ_c, $\hat{\eta}(t)$ is eliminated by cancellation of the contributions from the memory at $t-t'\gg 1/(\gamma H)$.

In conclusion, we derived the equation for the motion of magnetization in the presence of a spin current by using the local equilibrium assumption in nonequilibrium thermodynamics. We demonstrated that the value of the coefficient β is not equal to that of the Gilbert damping constant α in general. However, we also show that the equality $\alpha=\beta$ holds if $\tau_c\ll 1/(\gamma H)$. We then applied our theory to current-induced magnetization reversal in magnetic multilayers and showed that the switching time is a decreasing function of τ_c.

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[1] J. C. Slonczewski, J. Magn. Magn. Mater. **159**, L1 (1996).
[2] L. Berger, Phys. Rev. B **54**, 9353 (1996).
[3] J. A. Katine, F. J. Albert, R. A. Buhrman, E. B. Myers, and D. C. Ralph, Phys. Rev. Lett. **84**, 3149 (2000).
[4] M. Kläui, C. A. F. Vaz, J. A. C. Bland, W. Wernsdorfer, G. Faini, E. Cambril, and L. J. Heyderman, Appl. Phys. Lett. **83**, 105 (2003).
[5] S. I. Kiselev, J. C. Sankey, I. N. Krivorotov, N. C. Emley, R. J. Schoelkopf, R. A. Buhrman, and D. C. Ralph, Nature (London) **425**, 380 (2003).
[6] L. Landau and E. Lifshitz, Phys. Z. Sowjetunion **8**, 153 (1935).
[7] T. L. Gilbert, Armour Research Foundation Project No. A059, 1959 (unpublished); see also T. L. Gilbert, IEEE Trans. Magn. **40**, 3443 (2004).
[8] A. Thiaville, Y. Nakatani, J. Miltat, and Y. Suzuki, Europhys. Lett. **69**, 990 (2005).
[9] M. Viret, A. Vanhaverbeke, F. Ott, and J. F. Jaquinot, Phys. Rev. B **72**, 140403(R) (2005).
[10] J. He, Z. Li, and S. Zhang, Phys. Rev. B **73**, 184408 (2006).
[11] S. E. Barnes and S. Maekawa, Phys. Rev. Lett. **95**, 107204 (2005).
[12] H. Kohno, G. Tatara, and J. Shibata, J. Phys. Soc. Jpn. **75**, 113706 (2006).
[13] Y. Tserkovnyak, H. J. Skadsem, A. Brataas, and G. E. W. Bauer, Phys. Rev. B **74**, 144405 (2006).
[14] Y. Tserkovnyak, A. Brataas, and G. E. W. Bauer, J. Magn. Magn. Mater. **320**, 1282 (2008).
[15] J. Xiao, A. Zangwill, and M. D. Stiles, Phys. Rev. B **73**, 054428 (2006).
[16] M. D. Stiles, W. M. Saslow, M. J. Donahue, and A. Zangwill, Phys. Rev. B **75**, 214423 (2007).
[17] R. A. Duine, A. S. Nunez, J. Sinova, and A. H. MacDonald, Phys. Rev. B **75**, 214420 (2007).
[18] S. Zhang and Z. Li, Phys. Rev. Lett. **93**, 127204 (2004).
[19] J. Zhang, P. M. Levy, S. Zhang, and V. Antropov, Phys. Rev. Lett. **93**, 256602 (2004).
[20] A. A. Tulapurkar, Y. Suzuki, A. Fukushima, H. Kubota, H. Maehara, K. Tsunekawa, D. D. Djayaprawira, N. Watanabe, and S. Yuasa, Nature (London) **438**, 339 (2005).
[21] H. Kubota, A. Fukushima, K. Yakushiji, T. Nagahama, S. Yuasa, K. Ando, H. Maehara, Y. Nagamine, K. Tsunekawa, D. D. Djayaprawira, N. Watanabe, and Y. Suzuki, Nat. Phys. **4**, 37 (2008).
[22] K. Miyazaki and K. Seki, J. Chem. Phys. **108**, 7052 (1998).
[23] A. Kawabata, Prog. Theor. Phys. **48**, 2237 (1972).
[24] S. R. de Groot and P. Mazur, *Nonequilibrium Thermodynamics* (North-Holland, Amsterdam, 1962).
[25] H. Ohno, Science **281**, 951 (1998).
[26] T. Dietl and H. Ohno, Mater. Today **9**, 18 (2006).